




CENTRO DI RICERCA MATEMATICA
ENNIO DE GIORGI

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Probabilistic Methods of Investigating
Interior Smoothness of
Harmonic Functions
Associated with
Degenerate Elliptic Operators

SCUOLA NORMALE SUPERIORE
PISA



Foreword

In May 2004 Nicolai Krylov (University of Minnesota) visited the Centro di Ricerca Matematica Ennio De Giorgi as UMI - Lecturer with the partial support of the Unione Matematica Italiana. It is a great pleasure to see now the enlarged version of his lectures to appear in the series of Publications of the Center.

Mariano Giaquinta

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